Geometric Modeling
Assignment sheet #7
"Blossoming/Polar Forms"
(due June 18th 2012 before the lecture)

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Exercise 1 (Convex Sets): [6 points]

There are two definitions of the convex hull of a point set $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^d$:

i. All convex combinations of points from P (reminder: convex combinations are linear combinations with non-negative weights that sum to one).

ii. A minimal convex set with respect to set inclusion "$\subseteq$" that contains P. A set C is convex if and only if it contains all straight line segments between any two points from C.

Prove that the two definitions are equivalent.

Hint: Consider two sets C_1 and C_2 that are the convex hull of P with respect to definition (i) and (ii) and show separately that $C_1 \subseteq C_2$ and $C_2 \subseteq C_1$, by contradiction.

Exercise 2 (De Casteljau algorithm and subdivision): [1+3+2 points]

Given the cubic polynomial curve

$$P(u) = -\binom{7/8}{5/8}u^3 + \binom{9}{15/4}u^2 - \binom{57/2}{9/2}u + \binom{30}{-1}$$

a. Find the polar form $p(u_1, u_2, u_3)$ of $P(u)$, as well as the Bézier points (the vertices of the control polygon) P_0,P_1,P_2,P_3 of $P(u)$ w.r.t. the interval [2,4]. Sketch the the control polygon. (Use a full A4 paper and a meaningful scale, like 4 cm = 1 unit)

b. Evaluate the polynomial $P(u)$ using the De Casteljau algorithm at the sample points $u \in \{5/2,\ 3, 7/2\}$ and draw it into the same graph.

c. Use the result from (b) for subdividing $P(u)$ at $u = 3$ and subdivide the right part of the curve again at its midpoint $u = 7/2$. Add this control polygon to the same graph like before and draw the curve described by $P(u)$.

Exercise 3 (Polar forms and derivatives): **[1+2 points]**

Given is the cubic polynomial curve

$$F(u) = \begin{pmatrix}15\\-6\end{pmatrix}u^3 + \begin{pmatrix}27\\10\end{pmatrix}u^2 - \begin{pmatrix}9\\9\end{pmatrix}u$$

w.r.t. the parameter interval [0,1].

a. Find the first and second derivative of F.

b. Find the polar form $f(u_1, u_2, u_3)$ of F as well as the polar forms of the derivatives F' and F''. Show that they are equal to $3f(u_1, u_2, \vec{1})$ and $6f(u_1, \vec{1}, \vec{1})$ respectively.

Note: $f(u_1, u_2, \vec{1})$) is short for $f(u_1, u_2, 1) - f(u_1, u_2, 0)$

Exercise 4 (DeBoor algorithm): **[3+2 points]**

Given the uniform B-spline defined by the points

$$P_o = \begin{pmatrix}-2\\-10\end{pmatrix},\ P_1 = \begin{pmatrix}-4\\2\end{pmatrix},\ P_2 = \begin{pmatrix}6\\5\end{pmatrix},\ \ P_3 = \begin{pmatrix}4\\-7\end{pmatrix}$$

and the knot vector [0,1,2,3,4,5].

a. Evaluate the position of the curve at parameter t=2.5 using DeBoor's algorithm. Sketch the control polygon and the points constructed by the algorithm.

b. For the B-spline from (a), compute the corresponding Bézier control points which describe the same cubic curve. Draw the points and the resulting Bézier curve.